PASSIVE FREQUENCY STANDARD: STABILITY VS QUANTIZATION NOISE IN A NON-MULTIPLE TRANSFORMATION CIRCUIT TO GENERATE RF EXCITATION SIGNAL

A..Gevorkyan, O.Kharchev
The Russian Institute of Radionavigation and Time
193124, 2 Rastrelli sq., St.Petersburg, Russia
e-mail adress: office @ rirt.ru

ABSTRACT

Given are the analytical research results for the quantization noise effect in a non-multiple transformation circuit which produces the excitation RF signal in the passive frequency standard on its stability. The noise of quantization are caused by a digital synthesizer with flows of multilevel signals to perform a step adjustment of output frequency at a level of about $(1\div10)\cdot10^{-13}$. As a result of this research a method of accounting the contribution of phase noise due to quantization in the output frequency stability is proposed using Allan variance version.

Keywords: Frequency standard, non-multiple transformation, quantization noise, stability.

1. INTRODUCTION

Concerning the cesium-beam frequency standard the basic contribution of phase noise in short-term stability of output frequency due to quantization takes place. The more the noise level of quantization and the broader the frequency band covered by given noise the more is instability of output frequency. The results of researches are submitted as the curves describing dependence of instability caused by noise of excitation RF signal on measurement duration time in case of amplitude and phase mixing. The best results are obtained in case of phase mixing.

2. STABILITY VS QUANTIZATION NOISE

The principle of operation of passive frequency standard (PFS) is well known – Ref,1. To produce the excitation RF signal with frequency corresponds to clock transition a non-multiple transformation circuit is used – Ref, 2,3. In this case in structure of frequency converter the frequency of adjusted quartz oscillator f_1 is multiplied by an integer number N and result frequency is added to the output frequency of synthesizer f_2 as a product of amplitude or phase modulation so that equality is carried out

$$f_0 = f_1 N + f_2 \qquad (1)$$

The frequency converter (Fig.1) consist of series connected preliminary multiplier 1, mixer 3 (modulator) and final frequency multiplier 5. The second input of mixer 3 is connected to the input of preliminary multiplier though series connected the bandpass filter 4 and frequency synthesizer 2. The input signal with frequency f_1 is applied to the input of preliminary frequency multiplier 1 with a multiplication factor k_1 and to the input of frequency synthesizer 2 with coefficient of frequency transformation equal k_3.

Output signals of preliminary multiplier and synthesizer after filtering are mixed or modulated in mixer 3. The useful side spectral component of output signal from mixer is filtered and its frequency multiplied by final multiplier 5 by k_2 times. Thus total factor of multiplication is equal

$$N = k_1 k_2 , \qquad (2)$$

and total factor of frequency conversion is equal

$$N_1 = \frac{f_0}{f_1} = k_1 k_2 + k_3 . \qquad (3)$$

The accuracy of output signal from converter as well as the actual meaning of standard frequency is determined by an adjusting step of synthesizer which has an amount of this step about $(1\cdot10^{-14}\div1\cdot10^{-12})$. The output signal of synthesizer enriched by flows of multilevel signals as a product of synthesis, to filter which is practically impossible. However, by means of bandpass filter 4, one can restrict the band of passing frequencies. In case of digital synthesizer with flows of multilevel signals the noise level can be evaluated by a level of nearby noise spurious components, or noise level of quantization. Neglecting of instant meanings quantization noise and limiting only by noise of phase quantization, the level of phase noise is defined by expression – Ref, 3

$$D_n = 20\lg\frac{\pi}{\sqrt{12}\,2^{k+1}} , \qquad (4)$$

where k – digit in phase quantization. For obtaining $D_n \le 60$dB its enough to have k=9. Each additional digit of phase quantization reduced phase noise by 6dB, but requires double increase in device memory. The analysis of phase noise influence due to quantization in the output frequency stability of standard is of interest.

The converter circuit given on a fig.1 allows to analyze the process of non-multiple transformation in case of amplitude modulation and phase modulation. In case of amplitude modulation the frequency multiplication factors are equal to k_1=N, k_2=1 and as a mixer 3 can be used the balance modulator. In case of phase modulation the final step of frequency multiplier, usual a generator of harmonics on the diode with charge accumulation – Ref, 2 is used as a mixer.

In this case the factor of frequency multiplication given by final step 5 is much more than 1 and the equality (2) and (3) are valid.

Let us assume that noise spectral density of input signal of frequency converter – Ref, 4 i.e. output signal of adjusted quartz oscillator (q.o.) is defined as a flicker noise

$$S_g(\Omega) = \frac{A}{\Omega} , \qquad (5)$$

where A – level of phase noise; Ω - frequency.

Then spectral density of phase noise at the output of preliminary multiplier 1 is equal:

$$S_1(\Omega) = k_1{}^2 \frac{A}{\Omega}, \quad (6)$$

and at the output of synthesizer 2 after filtering by bandpass filter 4 is described by following expression:

$$S_2(\Omega) = k_3^2 \frac{A}{\Omega} + D, \quad (7)$$

where D – noise of phase quantization of synthesizer

$$D = \begin{cases} D_1 \text{ at } \Omega_1 \le \Omega \le \Omega_2 \\ 0 \text{ at } 0 < \Omega < \Omega_1 \text{ and } \Omega > \Omega_2 \end{cases},$$

$2\Omega_2$ a filter 4 passband in the assumption of the ideal bandpassing filter.
Ω_1 – lower value of analysis frequency, Ω_1>0.

For the square-law mixer 3 with transfer factor K_d at amplitude and phase non-multiple transformation of frequency spectral density of output signal phase noise is described by the following expression – Ref, 4,5,

$$S_3(\Omega) = 2K_d^2 \cdot [S_{11}(\Omega) + S_{12}(\Omega) + S_{22}(\Omega)], \quad (8)$$

where $S_{11}(\Omega)$ and $S_{22}(\Omega)$ autoconvolutions of preliminary multiplier 1 and synthesizer 2 output signals with bandpass filter 4 correspondingly; $S_{12}(\Omega)$ spectral density of phase noise caused by interaction of a signal of preliminary multiplier and synthesizer

$$S_{12}(\Omega) = \int_{\Omega_1}^{\infty} [S_1(\xi) S_2(\Omega - \xi) + S_2(\xi) S_1(\Omega - \xi)] d\xi \quad (9)$$

As a result of substitution of expressions (6), (7) and calculation the integrals in (8) and (9) limited from below by frequency Ω_1 the following expression of phase noise spectral density at the mixer output is received:

$$S_3(\Omega) = 2K_d^2 \Big\{ A\Big[\frac{A}{\Omega} \left(k_1^4 + 2k_1^2 k_3^2 + k_3^4\right) + \left(k_1^2 + k_3^2\right) D_1 \Big] \ln\left| \frac{\Omega}{\Omega_1} - 1 \right| + D_1^2 (\Omega_2 - \Omega_1) \Big\}. \quad (10)$$

From here spectral density of phase noise at the output of the final multiplier 5 and whole frequency is given by

$$S_5(\Omega) = k_2^2 S_3(\Omega). \quad (11)$$

And spectral density of PFS fluctuations in account of phase lock loop operation is given by

$$S(\Omega) = \left(K(p)\right(^2 \Omega^2 S_5(\Omega), \quad (12)$$

where

$$|K(p)|^2 = \frac{\left(\frac{1}{N}\right)^2}{1 + \left(\frac{\Omega}{\Omega_0}\right)^2}$$ - a module square of frequency fluctuations transfer factor of converter output signal in a loop,

$$\Omega_0 = \frac{N \cdot A_d K_g K_0}{T}$$ - cut-off frequency of adjusting circuit,
A_d – the slope of a quantum discriminator,
K_g – the slope of the q.o. adjusting range,
K_o and T gain and time constant of the integrating amplifier.

The received expression allows to determine instability of passive standard output frequency as Allan variance version – Ref, 6.

$$\sigma(\tau) = \frac{1}{\omega_0} \sqrt{\frac{1}{\pi} \int_0^{\infty} \frac{\sin^4\left(\frac{\Omega\tau}{2}\right)}{\left(\frac{\Omega\tau}{2}\right)^2} S(\Omega) d\Omega}, \quad (13)$$

where ω_0 – frequency of q.o., τ - measurement time.

The given expression allows to determine a contribution in total instability concerning PFS caused by noise of output signal of the non-multiple frequency converter, taking into account the noise level and band width quautization of phase, level of the q.o. own noise and in assumption of statistical independence the noise of preliminary multiplier signal and synthesizer signal.
As examples in Fig.2 and 3 the Allan variance vs measurement time for C_s atomic-beam standard are submitted at following assumption: frequency of q.o. 5MHz, N=1836, Ω_0=10Rad/s, $\Omega_1=10^{-7}$Rad/s, the synthesizer output frequency 12,6MHz.
The similar dependencies are true for Rb and H_2 PFS.

In Fig.2 the Allan variance for a case of amplitude mixing is submitted. In Fig.2a at several values of phase quantozation noise. In Fig.2b at several values of q.o. noise.

In a Fig.3 the similar dependencies of Allan variance for a case of phase mixing are submitted. In a Fig.3a at several values of noise level of a phase quantization. In Fig.3b at several values of Ω_2 frequency, describing a band of phase quantization noise.

From figures it is seen that the noise of phase quantization brings into instability of PFS output signal the contribution at short measurement time. And this contribution is more the more the noise level of quantization and the more wider frequency band occupied by the given noise.

In addition the comparison of frequency instability in case of amplitude and phase mixing specifies preferability of phase mixing.

CONCLUSION

Thus as a result of this research the method is offered for accounting the contribution of phase noise due to quantization of a phase in a circuit of non-multiple transformation of frequency in the output frequency of PFS. A preferability is demonstrated to use a phase mixing compared to amplitude mixing.

REFERENCES

1. V.V.Grigoriyanz, M.E.Gabotinsky, V.F.Zolin. Kvantovie standarty chastoty. Jurnal “Nauka”. 1968 in Russia.
2. V.Laptev, M.Kolosov etc. Izvestia Vusov. Priborostroenie. 1976, v.19, №13 in Russia.
3. D.Chapize, A.Panin. Osnovy teoriy sinteza chastot. Jurnal “Radio i Sviaz”. 1981 in Russia.
4. U.Evsikov, V.Chapursky. Preobrazovanie sluchainyh processov v radiotechnicheskih ustroistvah. Jurnal “Vishaia chcola”. 1977 in Russia.
5. B.Levin. Teoreticheskie osnovy statisticheskoy radiotechniky. Jurnal “Sov.radio”. 1969 in Russia.
6. J.A.Barnes, A.R.Chi, L.S.Cutler., IEEE Trans.Instrum.Meas., 1971, v.1M-20, May, pp.105-120.

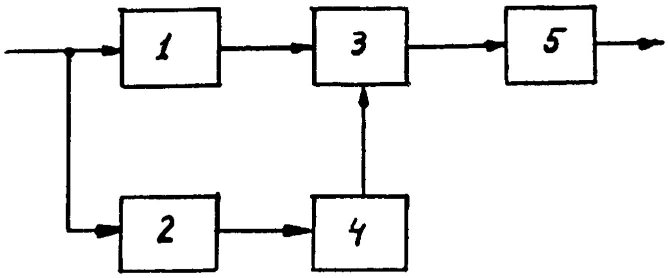

Fig.1. Frequency converter
1- multiplier, 2 – synthesizer, 3 – mixer, 4 – bandpass filter, 5 - multiplier

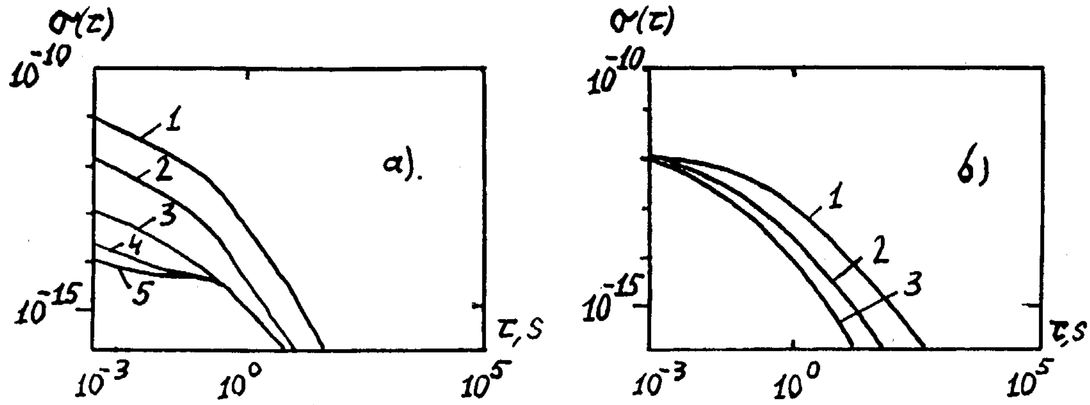

Fig.2. Allan variance using amplitude mixing
(k_1=1836, k_2=1)
a) A=5·10^{-12}, Ω_2=10^{3}Rad/s and D_1: 1 - 10^{-4}, 2 - 10^{-5}, 3 - 10^{-6}, 4 - 10^{-7}, 5 - 10^{-8}.
b) D_1=10^{-5}, Ω_2=10^{3}Rad/s and A: 1 - 5·10^{-10}, 2 - 5·10^{-11}, 3 - 5·10^{-12}.

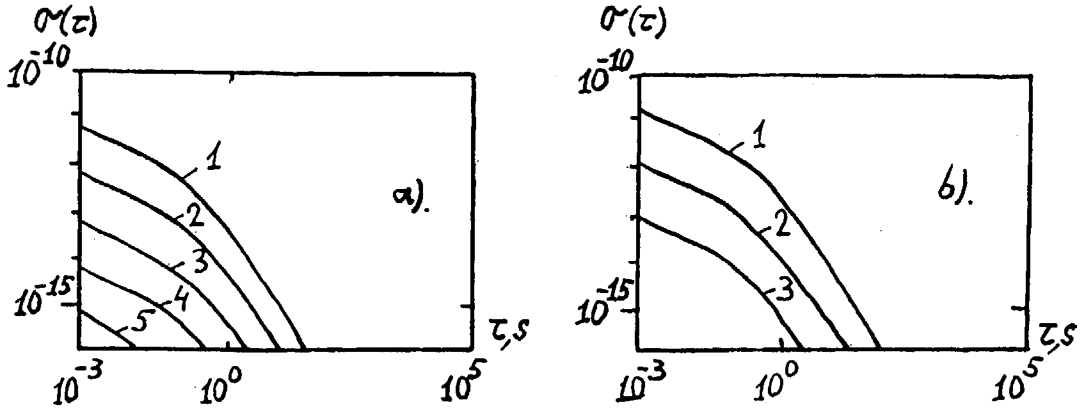

Fig.3. Allan variance in case of phase mixing
(k_1=54, k_2=34)
c) A=5·10^{-12}, Ω_2=10^{3}Rad/s and D_1: 1 - 10^{-4}, 2 - 10^{-5}, 3 - 10^{-6}, 4 - 10^{-7}, 5 - 10^{-8}.
d) A=5·10^{-12}, D_1=10^{-5}, Ω_2Rad/s: 1 - 5·10^{-5}, 2 - 5·10^3, 3 - 10.